At St Michael's we are using the 'White Rose Hub' format as a basis for our planning. We are not following it completely but use it as a tool.

We are using the White Rose Hub philosophy of:

  - **fluency** - using Learning Objectives from the National Curriculum

  - **reasoning**

  - **problem-solving**

In all our maths work we are using **a CPA approach** within our maths lessons (**CPA – Concrete/ Pictorial/ Abstract**)

We are using resources such as - White Rose, I See maths, NCETM Mastery documents & nrich problems.

The aim is that when children leave St Michael's they:

- Have a secure knowledge of number facts and a good understanding of the four calculation operations (addition, subtraction, multiplication and division)
- Make use of jottings, diagrams and informal notes to help record steps and part answers when using mental methods that generate more information than can be kept in their heads
- Have an efficient, reliable, written method of calculation for each operation that they are able to apply with confidence when they are unable to perform a calculation mentally

Maths Mastery

At the centre of the mastery approach to the teaching of maths is the belief that all children have the potential to succeed. They should have access to the same curriculum content and, rather than being extended with new learning, they should deepen their conceptual understanding by tackling challenging and varied problems. Similarly, with calculation strategies, children must not simply rote learn procedures but demonstrate their understanding of these procedures through the use of concrete materials and pictorial representations. This policy outlines the different calculation strategies that should be taught and used across the school, which is in line with the requirements of the 2014 Primary National Curriculum.

Mathematical Language

The 2014 National Curriculum is explicit in articulating the importance of children using the correct mathematical language as a central part of their learning (reasoning). In certain year groups, the non-statutory guidance highlights the requirement for children to extend their language around certain concepts. It is therefore essential that teaching using the strategies outlined in this policy is accompanied by the use of appropriate and precise mathematical vocabulary. New vocabulary should be introduced in a suitable context (for example, with relevant, real objects, apparatus, pictures of diagrams) and explained carefully. High expectations of the mathematical language used are essential, with teachers only accepting what is correct

This policy has been designed to teach children through the use of concrete, pictorial and abstract methods. This calculation policy should be used to support children to develop a deep understanding of number and calculation.

Using the Concrete-Pictorial-Abstract Approach:

Children develop an understanding of a mathematical concept through the three steps of: concrete, pictorial and abstract approach. Reinforcement is achieved by going back and forth between these representations.

Concrete Representation:

This is the first step in a child's learning. The child is introduced to an idea or skill by acting it out with real objects. This is a 'hands on' component using real objects and it is the foundation for conceptual understanding.

Pictorial Representation:

Once the child has sufficiently understood the 'hands on' experience, they can be progressed onto relating them to pictorial representations, such as a diagram or a picture of the problem.
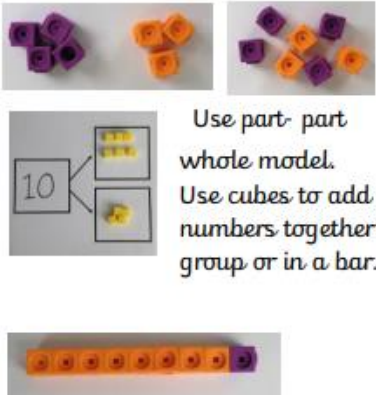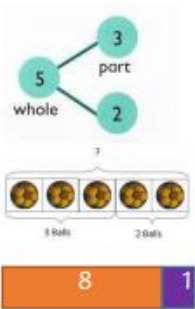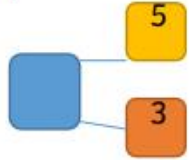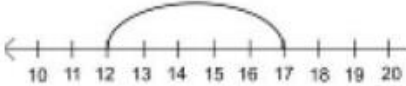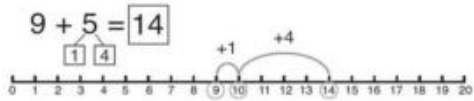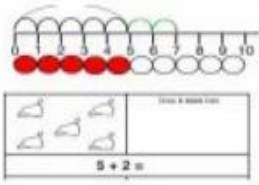
Abstract Representation:
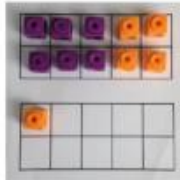
This is the third step in a child's learning. The child should now be capable of representing problems by using mathematical notation, for example: $12 \div 2 = 6$
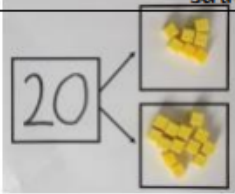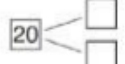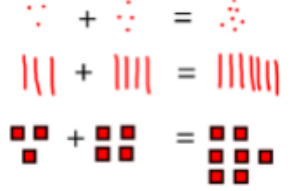
ADDITION
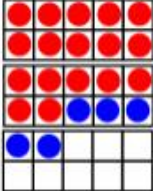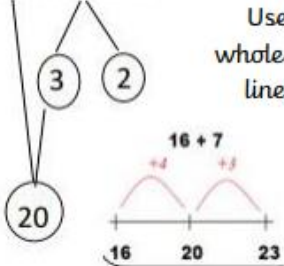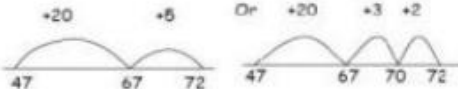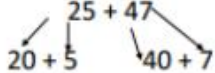
Year 1

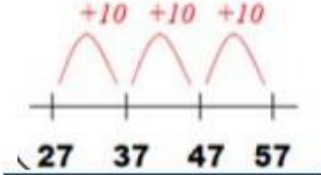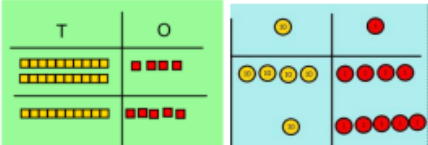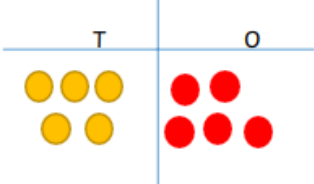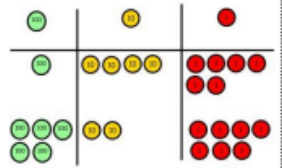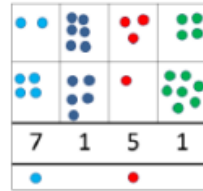| Objective & Strategy	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part-whole model.	Use part- part whole model. Use cubes to add two numbers together as a group or in a bar.	Use pictures to add two numbers together as a group or in a bar.	$4 + 3 = 7$ $10 = 6 + 4$ Use the part-part whole diagram as shown above to move into the abstract.
Starting at the bigger number and counting on.	Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	$12 + 5 = 17$ Start at the larger number on the number line and count on in ones or in one jump to find the answer.	$12 + 5 = 17$ Place the larger number in your head and count on the smaller number to find your answer.

Regrouping to make 10.	$6 + 5 = 11$ Start with the bigger number and use the smaller number to make 10.	$3 + 9 =$ Use pictures or a number line. Regroup or partition the smaller number to make 10. $9 + 5 = \boxed{14}$	$7 + 4 = 11$ "If I am at seven, how many more do I need to make 10? How many more do I add on now?"
Represent & use number bonds and related subtraction facts within 20.	2 more than 5.	$5 + 2 =$	Emphasis should be on the language: "1 more than 5 is equal to 6" "2 more than 5 is 7" "8 is 3 more than 5"

Year 2

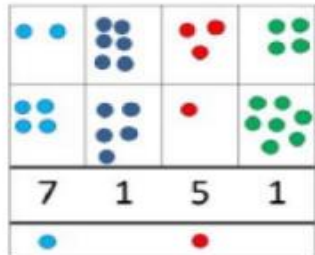| Objective & Strategy	Concrete	Pictorial	Abstract
Adding multiples of ten.	50 = 30 + 20 Model using dienes and bead strings.	3 tens + 5 tens = _____ tens 30 + 50 = _____ Use representations for base ten.	20 + 30 = 50 70 = 50 + 20 40 + ___ = 60
Use known number facts including different combinations of tens & ones of any 2 digit number. (Part part whole)	20 Children explore ways of making numbers.	20 □ + □ = 20 20 − □ = □ □ + □ = 20 20 − □ = □	Include teaching of the inverse of addition and subtraction: □ + 1 = 16 16 − 1 = □ 1 + □ = 16 16 − □ = 1
Use known facts.	□□□ + □□□ = □□□□□□ ▯▯▯ + ▯▯▯ = ▯▯▯▯▯▯	∴ + ∵ = (dots) ||| + |||| = ||||||| ▪▪ + ▪▪ = ▪▪▪ Children draw representations of H, T & O.	3 + 4 = 7 Leads to 30 + 40 = 70 Leads to 300 + 400 = 700
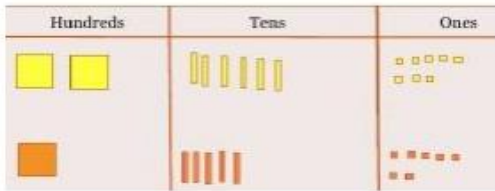
Use bar models.	 3 + 4 = 7	 7 + 3 = 10	23 | 25 ? 23 + 25 = 48
Add a two digit number and ones.	 17 + 5 = 22 Use ten frame to make 'magic ten'. Children explore the patterns: 17 + 5 = 22 27 + 5 = 32	17 + 5 = 22 3 2 20 Use part part whole and number line to model. 16 + 7 +4 +3 16 20 23	17 + 5 = 22 Explore related facts: 17 + 5 = 22 5 + 17 = 22 22 − 17 = 5 22 − 5 = 17 22 17 | 5
Add 2 digit numbers and tens.	 25 + 10 = 35 Explore that the ones digit does not change.	27 + 30 +10 +10 +10 27 37 47 57	27 + 10 = 37 27 + 20 = 47 27 + ___ = 57
Add two 2-digit numbers.	 Model using dienes, place value counters and numicon.	+20 +5 Or +20 +3 +2 47 67 72 47 67 70 72 Use number line and bridge ten using part whole if necessary.	25 + 47 20 + 5 40 + 7 20 + 40 = 60 5 + 7 = 12 60 + 12 = 72

Add three 1-digit numbers.	$4 + 7 + 6 = 17$ Put 4 and 6 together to make 10. Add on 7. Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.	 Regroup and draw representation. $+$ $= 15$	$4 + 7 + 6 = \boxed{10} + \boxed{7}$ 10 $= \boxed{17}$ Combine the two numbers that make/bridge ten, then add on the third.
Rapid Recall (addition and subtraction)	- Bonds within 10 - Bonds within 20 - Bonds to 100 (multiples of 10) - Add single-digit to make a multiple of 10	Strategies Add/subtract 9, 19, 29… Partitioning - Add near doubles - Reorder - Count on/back in 10s	

Year 3

Objective & Strategy	Concrete	Pictorial	Abstract

Column Addition – no regrouping (friendly numbers)
 	
Add 2 or 3 digit numbers.
	24 + 15 =
	
	Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters.
		After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions.
		
		T O
			Add the ones first, then the tens, then the hundreds:
			
			  2 2 3
			+ 1 1 4
			———
			  3 3 7
			
			Children use the 'steps to success' to format their calculation:
			
			'Steps for Success'
			1. Write your calculation, label your digits and circle the operation.
			2. Check your operation, choose your method and set it up below. Remember to leave plenty of room for working out.
			3. Use the method to calculate the answer.
			4. Write the answer at the end of the calculation.

Column Addition – with regrouping.	Make both numbers on a place value grid.
	
	146
	+ 527
	
	Add up the units and exchange 10 ones for one 10.
		Children can draw a pictorial representation of the columns and place value counters to further support their learning and understanding.
		
		7 1 5 1
			Children follow the 'Steps to Success' to regroup and form the calculation correctly:
			
			  1 5 3
			+ 3₁ 6 2
			———————
			  5 1 5
			
			Don't forget, if you pass hundred, save it above the line and add it on later
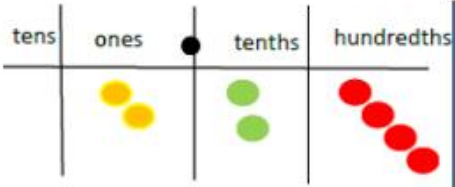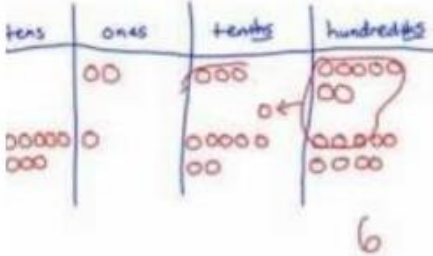
Year 4

Objective & Strategy	Concrete	Pictorial	Abstract
Year 4 Add numbers with up to 4 digits	Children continue to use dienes or place value counters to add, exchanging ten ones for a ten, ten tens for a hundred and ten hundreds for a thousand. Hundreds / Tens / Ones	Draw representations using place value grid. 7 1 5 1	Continue from previous work to carry hundreds as well as tens. Relate to money and measures.

In the Abstract column, a column addition:

$$\begin{array}{r} 4\ 2\ 2\ 2 \\ +\ 5_1\ 7_1\ 9\ 1 \\ \hline 1\ 0\ 0\ 1\ 3 \end{array}$$

Year 4 Rapid Recall (addition and subtraction)	• Sums/differences – multiples of 10/100/1000 • Doubles – within 100 • Add/subtract multiples of 10/100/1000	**Strategies** Partition Small difference • Bridging • Round & adjust
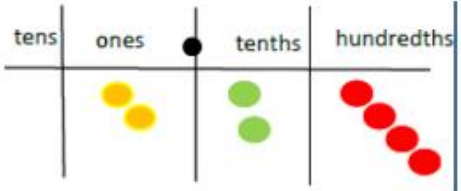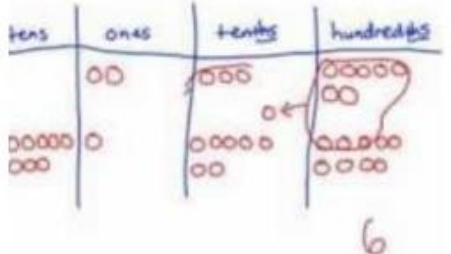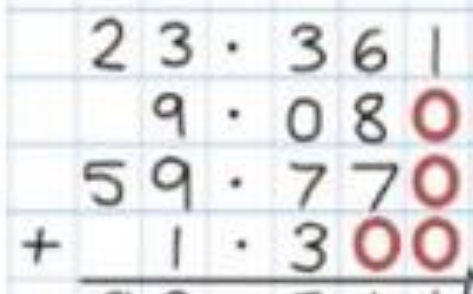
Year 5

Year 5 Add numbers with more than 4 digits. Add decimals with 2 decimal places, including money.	(As year 4) Introduce decimal place value counters and model exchange for addition. 	(As year 4) 2.37 + 81.79 	(As year 4)

For the third column:

$$
\begin{array}{r}
1\ 4\ 0\ 1\ .\ 2 \\
+\quad 1\ 2\ 6\ .\ 8\ 5 \\
1\ 0_1\ .\ 7\ 1 \\
\hline
1\ 5\ 3\ 8\ .\ 7\ 6
\end{array}
$$

Year 6
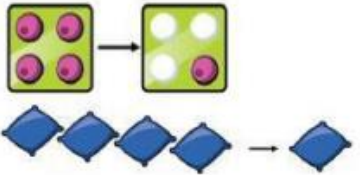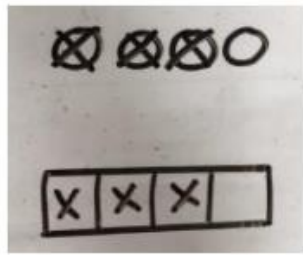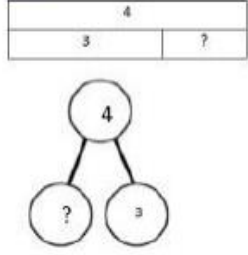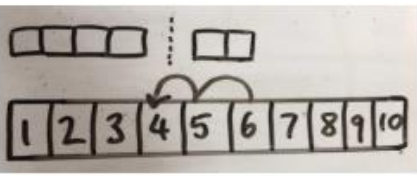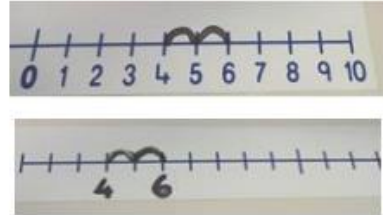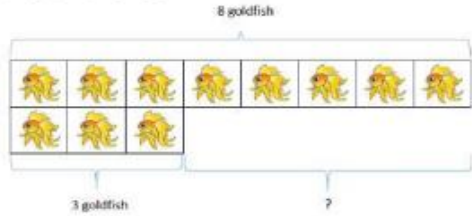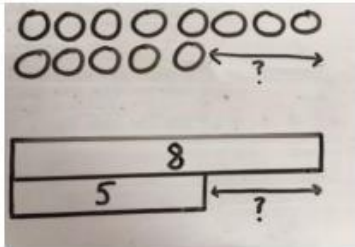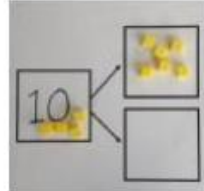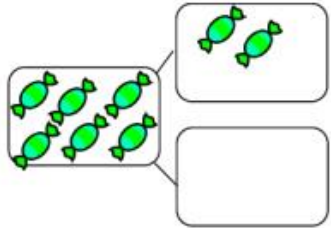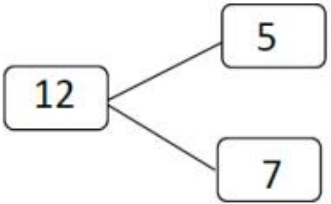
Year 6	(As year 5)	(As year 5)	Insert zeros for place holders.
Add several numbers of increasing complexity. Include adding money, measure and decimals with different numbers of decimal points.	Introduce decimal place value counters and model exchange for addition. tens | ones • tenths | hundredths	2.37 + 81.79	
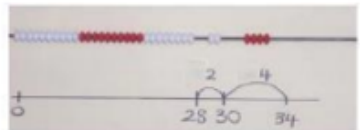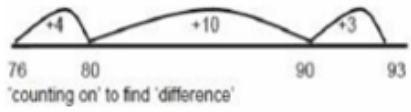
SUBTRACTION

Year 1
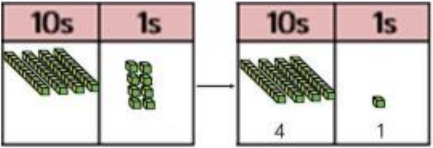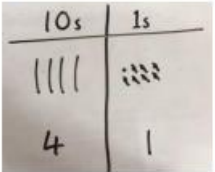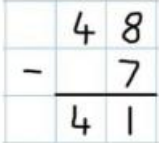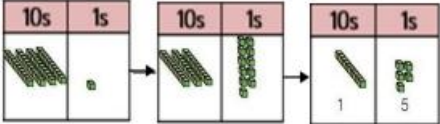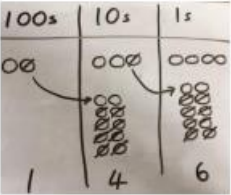
Objective & Strategy	Concrete	Pictorial	Abstract
Taking away ones from a whole.	Use physical objects, counters, cubes etc. to show how objects can be taken away. $4 - 3 = 1$	Cross out drawn objects to show how many has been taken away. The bar model can also be used.	$4 - 3 =$ $\square = 4 - 3$
Counting back.	Counting back (using number lines or number tracks) children start with 6 and count back 2. $6 - 2 = 4$	Children to represent what they see pictorially e.g.	Children to represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line.

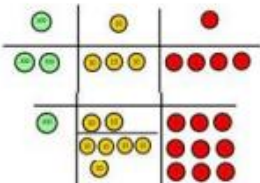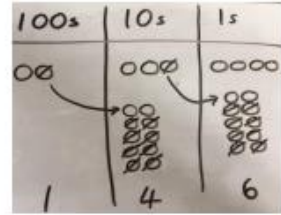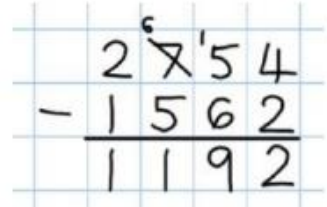| Finding the difference. | Compare amounts and objects to find the difference.

8 goldfish

3 goldfish ?

Use cubes to build towers or make bars to find the difference. Use basic bar models with items to find the difference. | Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate.

 | Find the difference between 8 and

5. 8 − 5, the differences is...

Children to explore why
9 − 6 = 8 − 5 = 7 − 4 have the same difference. |
| Represent and use number bonds and related subtraction facts within 20.

(Part part whole model) | Link to addition − use the PPW model to model the inverse.

If 10 is the whole and 6 is one of the parts, what is the other part?
10 − 6 = 4 | Use pictorial representations to show the parts.

 | Move to using numbers within the part whole model.

 |

Year 2

Objective & Strategy	Concrete	Pictorial	Abstract
Partitioning to subtract – without regrouping. (friendly numbers)	Use dienes to show how to partition the number when subtracting without regrouping. 34 – 13 = 21	Children draw representations of dienes and cross off. 43 – 21 = 22	43 – 21 = 22
Making ten. (crossing one ten, crossing more than one ten, crossing the hundreds)	Use a bead string to model counting to the next ten and the rest. 34 – 28 = 2 4 28 30 34	Use a number line to count on to the next ten and then the rest. +4 +10 +3 76 80 90 93 'counting on' to find 'difference'	93 – 76 = 17

Year 3

Objective & Strategy	Concrete	Pictorial	Abstract
Column subtraction without regrouping. (friendly numbers)	Column method using base ten.	Children to represent the base 10 pictorially.	Column method or children could count back 7. Children use their 'Steps to Success' to format the question correctly:
Column subtraction with regrouping.	Column method using base 10 and having to exchange. $41 - 26 =$	Represent the place value counters pictorially; remembering to show what has been exchanged.	Formal column method using 'Steps to Success'. Children must understand what has happened when they have crossed out digits.

Year 4

Objective & Strategy	Concrete	Pictorial	Abstract
Year 4 Subtracting tens and ones – up to 4 digits. (introduce decimal subtraction through context of money)	Model process of exchange using numicon, base ten and then move to place value counters. 234 – 179 = 	Represent the place value counters pictorially; remembering to show what has been exchanged. 	Formal column method. Children must understand what has happened when they have crossed out digits.

Year 5

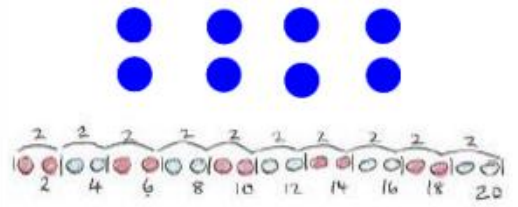| Year 5 Subtract with at least 4 digits, including money and measures. (subtract with decimal values, including mixtures of integers and decimals and aligning the decimal)	Model process of exchange using numicon, base ten and then move to place value counters. 234 – 179 =	Represent the place value counters pictorially; remembering to show what has been exchanged.	Formal column method. Children must understand what has happened when they have crossed out digits. Use zeros for place holders.
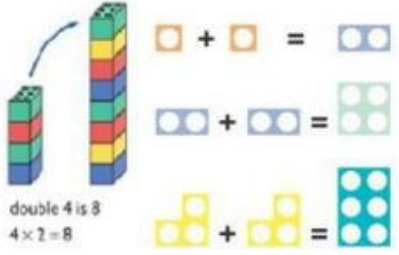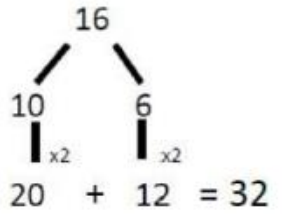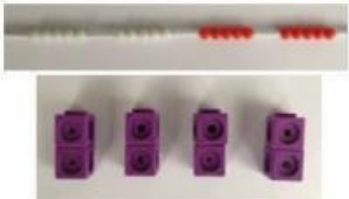
Year 6

Year 6 Subtract with increasingly large, more complex, numbers and decimal values.	Model process of exchange using numicon, base ten and then move to place value counters.	Represent the place value counters pictorially; remembering to show what has been exchanged.	Increasingly large and more complex numbers.
	$234 - 179 =$		

Multiplication

Year 1

Objective & Strategy	Concrete	Pictorial	Abstract
Doubling numbers.	Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling. double 4 is 8 $4 \times 2 = 8$	Draw pictures to show how to double numbers. Double 4 is 8	Partition a number and then double each part before recombining it back together. 16 10 6 x2 x2 20 + 12 = 32
Counting in multiples.	Count the group as children are skip counting, children may use their fingers to help.	Children make representations to show counting in multiples. 2 2 2 2 2 2 2 2 2 2 2 4 6 8 10 12 14 16 18 20	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25, 30

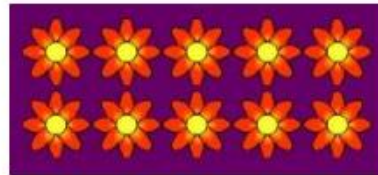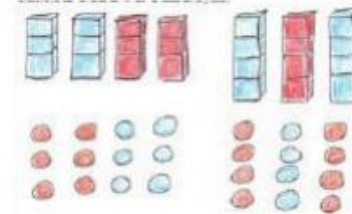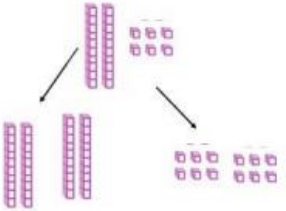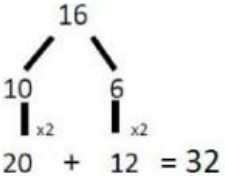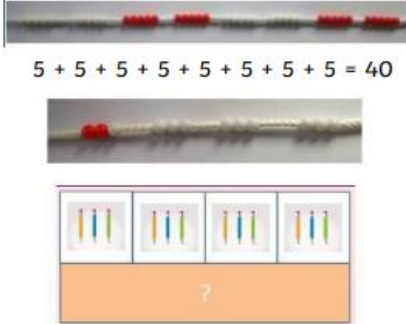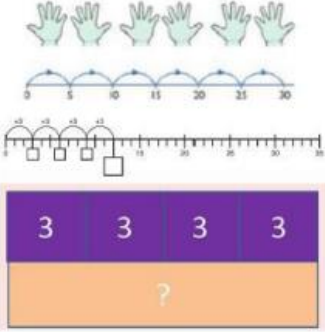| Repeated grouping/repeated addition.	3 x 4 = 4 + 4 + 4 = There are 3 equal groups, with 4 in each group.	Children to represent the practical resources in a picture and use a bar model.	3 × 4 = 12 4 + 4 + 4 = 12
Understanding arrays.	Use objects laid out in arrays to find the answers to 2 lots of 5, 3 lots of 2s.	Draw representations of arrays to demonstrate understanding.	3 x 2 = 6 2 x 5 = 10
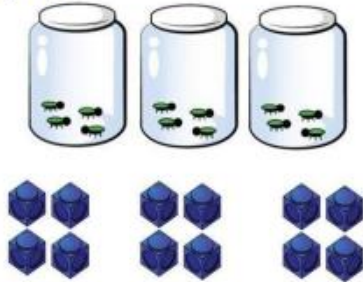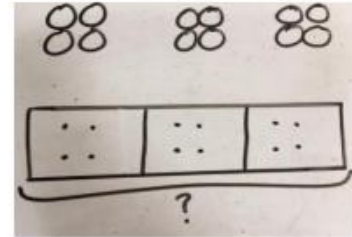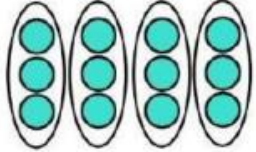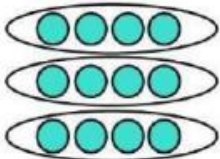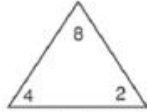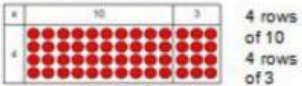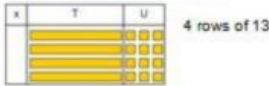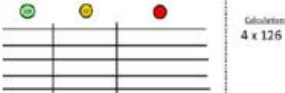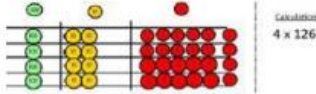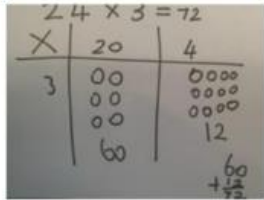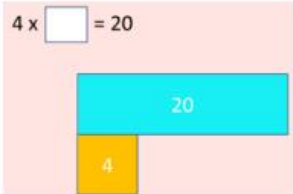
Year 2

Objective & Strategy	Concrete	Pictorial	Abstract
Doubling numbers.	Model doubling using dienes and place value counters. Doubling 26 	Draw pictures and representations to demonstrate how to double numbers	Partition a number and then double each part before recombining it back together. 16 10 6 x2 x2 20 + 12 = 32
Counting in multiples of 2, 5 and 10 from 0. (repeated addition)	Count the groups as children are skip counting, children may use their fingers to help. Progress onto bar models. 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40	Number lines, counting sticks and bar models should be used to show representation of counting in multiples. | 3	3	3	3
?					Count in multiples of a number aloud. Write sequences with multiples of numbers. 0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25, 30 4 x 3 = __

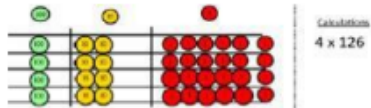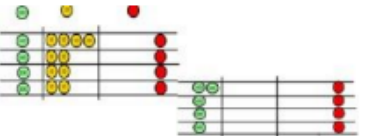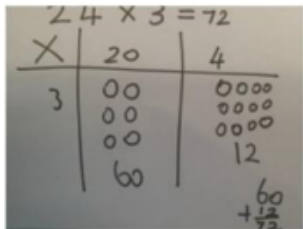|			
Multiplication is commutative.	Create arrays using counters, cubes and numicon. Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not change the answer.	Use representations of arrays to show different calculations and explore commutativity.	$12 = 3 \times 4$ $12 = 4 \times 3$ Use an array to write multiplication sentences and reinforce repeated addition. $5 + 5 + 5 = 15$ $3 + 3 + 3 + 3 + 3 = 15$ $5 \times 3 = 15$ $3 \times 5 = 15$
Using the inverse. (this should be taught alongside division, so pupils learn how the two operations work alongside each other)			$2 \times 4 = 8$ $4 \times 2 = 8$ $8 \div 2 = 4$ $8 \div 4 = 2$ $8 = 2 \times 4$ $8 = 4 \times 2$ $2 = 8 \div 4$ $4 = 8 \div 2$ Show all 8 related fact family sentences.

Year 3

Objective & Strategy	Concrete	Pictorial	Abstract
The grid method.	Show the links with arrays to first introduce the grid method. 4 rows of 10 4 rows of 3 Move onto base ten to move towards a more compact method. 4 rows of 13 Move onto place value counters to show how we are finding groups of a number. We are multiplying by 4, so we need 4 rows... Calculations 4 x 126 Fill each row with 126... Calculations 4 x 126 Add up each column, starting with the ones making any exchanges needed.	Children can represent their work with place value counters in a way that they understand. They can draw the counters using colour to show different amounts or just use the circles in the different columns to show their thinking. $24 \times 3 = 72$ Bar models are used to explore missing numbers. $4 \times \boxed{} = 20$ 20 4	Begin with multiplying by one digit numbers and showing the clear addition alongside. 	$\times$	30	5	 	7	210	35	 $210 + 35 = 245$ Moving forward, multiply by a 2 digit number, showing the different rows within the grid method. 		10	8	 	10	100	80	 	3	30	24	

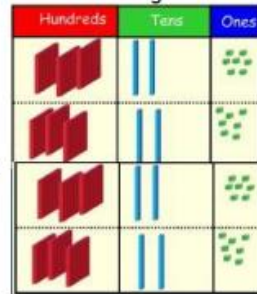|			
	 Then you have your answer.		
Rapid Recall (multiplication and division	• Multiplication and division facts for 2, 5, 10, 3, 4 and 8 times tables.	Strategies • 'Double-double'/'half-half' links within the listed times tables. • Associativtity (pushing numbers around) • Using what I already know	

Year 4

Objective & Strategy	Concrete	Pictorial	Abstract
The grid method (recap from Year 3 for 2 digit x 1 digit). Children progress to multiplying 3 digit numbers by 1 digit (Year 4 expectation).	Use place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows. 4 x 126 Fill each row with 126. Add up each column, starting with the ones making any exchanges needed.	Children can represent their work with place value counters in a way that they understand. They can draw the counters using colour to show different amounts or just use the circles in the different columns to show their thinking. 	Multiply 3 digit by 1 digit numbers using the grid method. <table><tr><td>x</td><td>300</td><td>20</td><td>7</td></tr><tr><td>4</td><td>1200</td><td>80</td><td>28</td></tr></table> 1200 + 80 + 28 = 1,308

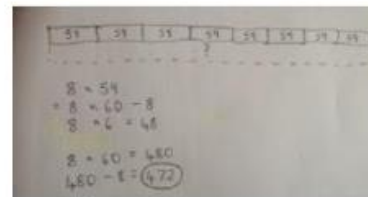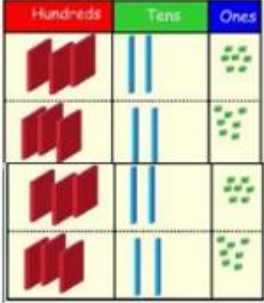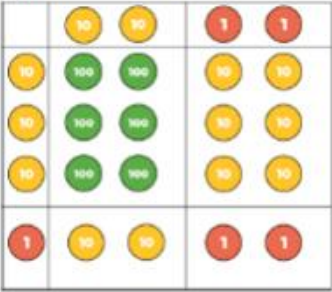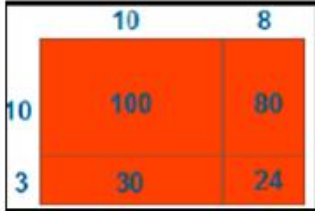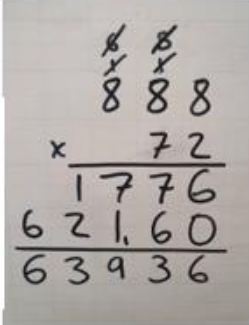| Column Multiplication. | Children can continue to be supported by place value counters at this stage of multiplication. This is initially done where there is no regrouping. 321 x 2 = 642 It is important at this stage that they always multiply the ones column first. The corresponding long multiplication is modelled alongside this method. | The grid method may be used to show how this relates to a formal written method (see abstract column). Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods. | The grid method can then be progressed onto the compact method. $$\begin{array}{r} 4\,^5\,7 \\ \times \qquad 8 \\ \hline 3\ 7\ 6 \end{array}$$ |

Grid method table:

x	300	20	7
4	1200	80	28

Year 5

Objective & Strategy	Concrete	Pictorial	Abstract
Column Multiplication (3 and 4 digits x 1 digit).	Children can continue to be supported by place value counters at this stage of multiplication. This is initially done where there is no regrouping.	The grid method may be used to show how this relates to a formal written method (see abstract column).	The grid method can then be progressed onto the compact method.
Column Multiplication – Long multiplication.	Manipulatives may still be used with the corresponding long multiplication modelled alongside. (22 x 31)	Continue to use bar modelling to support problem solving.	Progress to using the column method for long multiplication.

Rapid Recall (multiplication and division	• Square numbers to 144 • Establish whether a number is prime • Recall all prime numbers up to 19	Strategies • X by 9 • X/÷ by 10/100/1000 – including decimals • Use what you know to... • x/÷ by 5/50/25 • x by $\frac{1}{2}$ • Use factor pairs – 24 x 16

Year 6

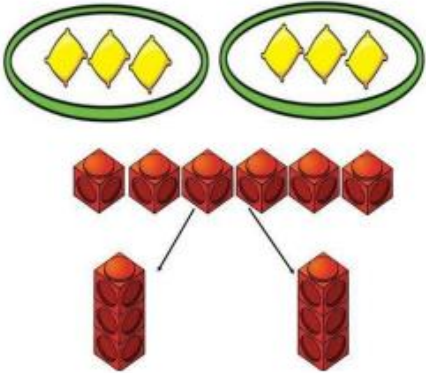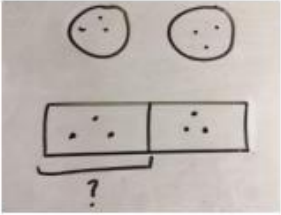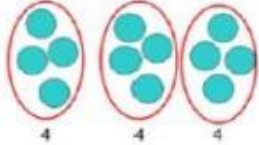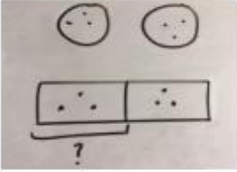| Objective & Strategy	Concrete	Pictorial	Abstract
Column Multiplication – Long multiplication.	Manipulatives may still be used with the corresponding long multiplication modelled alongside.	Continue to use bar modelling to support problem solving.	Progress to using the column method for long multiplication.
Multiplying decimals up to 2 decimal places by a single digit.			Remind children that the single digit belongs in the ones column. Line up the decimal points in the question and answer.

			When appropriate, children can use their place value knowledge to make the number being multiplied 10, 100 or 1000 times bigger and then multiply and make the answer 10, 100 or 1000 times smaller.

$$319_{(\times100)}$$
$$\times \quad 8$$
$$\overline{2552}_{(\div100)} = 25.52$$
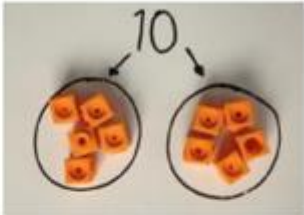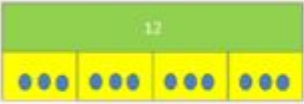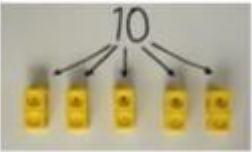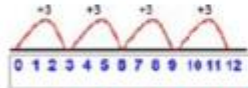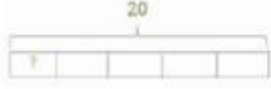
Division

Year 1

Objective & Strategy	Concrete	Pictorial	Abstract
Division as sharing	Sharing using a range of objects: 6 ÷ 2 = 	Use pictures or shapes to share quantities: Sharing: 4 4 4 12 shared between 3 is 4	Children continue with pictorial method until fully secure. Children should also be encouraged to use their 2 times tables facts. To progress further, children can then be moved onto: '6 shared between 2 is 3'

Year 2

Objective & Strategy	Concrete	Pictorial	Abstract
Division as sharing	I have 10 cubes, can you share them into 2 equal groups?	Children use pictures or shapes to share quantities: 8 ÷ 2 = 4 Children use bar modelling to show and support understanding: 12 ÷ 4 = 3	$12 \div 3 = 4$
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.	Use number lines for grouping: Use bar model to support with division: 20 ÷ 5 = ? 5 x ? = 20	$28 \div 7 = 4$ Divide 28 into 7 groups. How many are in each group?

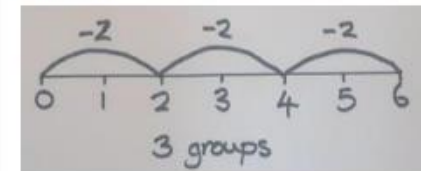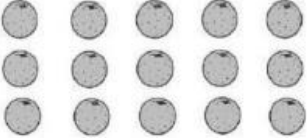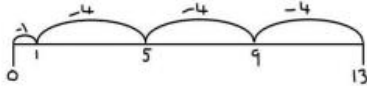| Division using repeated subtraction	Repeated subtraction using Cuisenaire rods above a ruler: 6 ÷ 2 3 groups of 2	Children to represent repeated subtraction pictorially:	Abstract number line to represent the equal groups that have been subtracted:
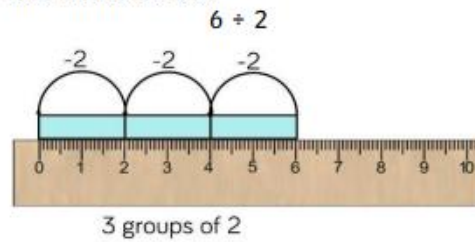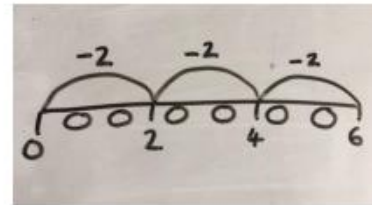
Year 3

Objective & Strategy	Concrete	Pictorial	Abstract
Division with arrays	Link division to multiplication by creating an array and thinking about the number sentences that can be created: $15 \div 3 = 5$ $5 \times 3 = 15$ $15 \div 5 = 3$ $3 \times 5 = 15$	Draw an array and use lines to split the array into groups to make multiplication and division sentences: $15 \div 3 = 5$ $5 \times 3 = 15$ $15 \div 5 = 3$ $3 \times 5 = 15$	Find the inverse of multiplication and division sentences by creating eight linking number sentences: $7 \times 4 = 28$ $4 \times 7 = 28$ $28 \div 7 = 4$ $28 \div 4 = 7$ $28 = 7 \times 4$ $28 = 4 \times 7$ $4 = 28 \div 7$ $7 = 28 \div 4$
Division with remainders	This can be done with lollipop sticks or Cuisenaire rods: $13 \div 4$ Use of lollipop sticks to form wholes-squares are made because we are dividing by 4. There are 3 whole squares, with 1 left over.	Children to represent the lollipop sticks pictorially: There are 3 whole squares, with 1 left over.	$13 \div 4 = 3$ remainder 1 Children should be encouraged to use their times table facts; they could also represent repeated addition on a number line: '3 groups of 4, with 1 left over'

Year 4- 6

Long division with remainder		
	Begin by modelling method with a 1-digit divisor. Write out the first 5 multiples (if needed continue). Find the closest number- How many times? $14 \times 1 = 14.$ Subtract Bring down the next digit Repeat until there is no digits left to bring down and create a remainder if necessary.	Divide- Put the number in the bus stop. Multiples- Write down the first 5 multiples Find the closest number- Look at the multiples and find the closest number. How many times- Count the multiples and add the number on top of the bus stop. Subtract Bring down the next number

Long division with decimal remainders		
	$$\begin{array}{r} 25.2 \\ 5\overline{)126.0} \\ -10 \\ \hline 26 \\ -25 \\ \hline 10 \\ -10 \\ \hline 0 \end{array}$$ When there is a remainder which you need to write as a decimal, bring down the 0 in the from then tenths column, and repeat the process as before.	